Three generation neutrino mixing is compatible with all experiments

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Abstract

We consider the minimal extension of the Standard Model with three generations of massive neutrinos that mix. We then determine the parameters of the model that satisfy all experimental constraints.
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Three observables in disagreement with the Standard Model of Quarks and Leptons are: i) A deficit of electron-type solar neutrinos; ii) A deficit of muon-type atmospheric neutrinos; and, possibly, iii) The observation of the apearance of $\bar{\nu}_e$ in a beam of $\bar{\nu}_\mu$ by the LSND Collaboration. The invisible width of the Z implies that the number of massless, or light Dirac, or light Majorana neutrino species is $N_\nu = 2.993 \pm 0.011$.[1] To account for these observations we consider the minimal extension of the Standard Model with three massive neutrinos that mix. The neutrino interaction eigenstates ν_l are superpositions of the neutrino mass eigenstates ν_m:

$$|\nu_l\rangle = \sum_m U_{lm}|\nu_m\rangle \tag{1}$$

We consider the "standard" parametrization of the unitary matrix U_{lm}[1]:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{2}$$

where $c_{ij} \equiv cos\theta_{ij}$, $s_{ij} \equiv sin\theta_{ij}$, $0 \leq \theta_{ij} \leq \frac{\pi}{2}$ and $-\pi \leq \delta < \pi$. The probability that an ultrarelativistic neutrino produced as ν_l decays as $\nu_{l'}$

1

Experiment	Energy [MeV]	Observed flux (SNU)	SSM prediction (SNU)	Ratio
Homestake[2]	0.87	2.56 ± 0.23	$7.7^{+1.2}_{-1.0}$	0.33 ± 0.05
Sage[3]	$0.233 - 0.4$	67 ± 8	129^{+8}_{-6}	0.52 ± 0.07
Gallex[4]	$0.233 - 0.4$	78 ± 8	129^{+8}_{-6}	0.60 ± 0.07
Kamiokande[5]	$7 - 13$	2.80 ± 0.38	$5.2^{+1.0}_{-0.7}$	0.53 ± 0.11
Super-Kam.[6]	$6 - 13$	$2.42^{+0.12}_{-0.09}$	$5.2^{+1.0}_{-0.7}$	0.46 ± 0.08

Table 1: Observed solar electron-type neutrino flux, compared to the Standard Solar Model (SSM) predictions asuming no neutrino mixing[7], and their ratio. The Solar Neutrino Unit (SNU) is 10^{-36} captures per atom per second. For Kamiokande (Super Kamiokande) the flux is in units of $10^6 \mathrm{cm}^{-2}\mathrm{s}^{-1}$ at Earth above 7MeV (6.5MeV).

is[1]:

$$P(\nu_l \rightarrow \nu_{l'}) = |\sum_m U_{lm} exp(-iLM_m^2/2E)U_{l'm}^*|^2 = P(\bar{\nu}_{l'} \rightarrow \bar{\nu}_l) \qquad (3)$$

where E and L are the energy and traveling distance of ν_l, and M_m is the mass of ν_m. We choose $M_1 \leq M_2 \leq M_3$. This extension of the Standard Model introduces six parameters: s_{12}, s_{23}, s_{13}, δ, and two mass-squared differences, e.g. $\Delta M_{21}^2 \equiv M_2^2 - M_1^2$ and $\Delta M_{32}^2 \equiv M_3^2 - M_2^2$. We vary these parameters to minimize a χ^2. This χ^2 has 14 terms obtained from the solar neutrino data summarized in Table 1, the atmospheric neutrino data shown in Table 2, and the LSND data[9]: $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \frac{1}{2}\sin^2(2\theta) = 0.0031 \pm 0.0013$ for $L[\mathrm{km}]/E[\mathrm{GeV}] = [P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)]^{1/2}/1.27 \cdot \Delta M^2[\mathrm{eV}^2] \approx 0.73$ (here $\sin^2(2\theta)$ corresponds to "large" ΔM^2, and ΔM^2 corresponds to $\sin^2(2\theta) = 1$, see discussion in [1]). Because one author[10] of the LSND Collaboration is in disagreement with the conclusion, and because the result has not been confirmed by an independent experiment, we multiply the error by 1.5 and take $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = 0.0031 \pm 0.0020$. We require that the astrophysical, reactor and accelerator limits be satisfied. The most stringent of these limits are listed in Table 3.

The χ^2 has 8 degrees of freedom (14 terms minus 6 parameters). Varying the parameters we obtain minimums of χ^2, a few of which are listed in Table 4. With 90% confidence the neutrino mass-squared differences lie within the dots shown in Figure 1. Note that one of the mass-squared differences is determined by the solar neutrino experiments and the other one by the atmospheric neutrino observations.

If neutrinos have a hierarchy of masses (as the charged leptons, up quarks,

L/E [km/GeV]	R_e	R_μ
10	1.20 ± 0.15	1.00 ± 0.15
100	1.20 ± 0.15	0.85 ± 0.12
1000	1.20 ± 0.15	0.70 ± 0.10
10000	1.20 ± 0.15	0.60 ± 0.08

Table 2: Ratio of the numbers of observed and predicted electron-type and muon-type neutrinos as a function of the flight length-to-energy ratio as measured by the Super-Kamiokande Collaboration.[8] Because of the uncertainty on the absolute neutrino flux and because R_e is observed to be independent of L/E, we divide the numbers in this table by 1.15 so that $R_e \approx 1$.

or down quarks), then the "upper island" in Figure 1 applies, and $M_3 \approx 0.07$eV, $M_2 \approx 10^{-5}$eV and $M_1 < M_2$, with large uncertainties.

Note in Table 1 that the ratio of the observed-to-predicted solar neutrino flux is significantly lower for the Homestake experiment than for Sage, Gallex, Kamiokande and Super-Kamiokande which are all compatible with 0.5. These latter experiments observe neutrinos within wide energy bands, while the chlorine detector in the Homestake mine observes monochromatic electron-type neutrinos from a ^7Be line. For the Homestake experiment the spread in L/E is due to the spread in L, which in turn is due to the excentricity of the orbit of the Earth. Therefore the interference is coherent for up to $L\Delta M^2/(2E \cdot 2\pi) \approx 30$ oscillations from the Sun to the Earth (here ΔM^2 is either ΔM_{21}^2 or ΔM_{32}^2). Due to this coherence at "small" ΔM^2 it is possible to find acceptable solutions with $\chi^2 < 13.4$ as shown in Figure 1. For larger values of ΔM^2 coherence is lost and we find solutions with $\chi^2 > 18$ which are unacceptable if the Homestake experimental and theoretical errors are correct.

An important test of the model would be to observe seasonal variations of the neutrino flux of the ^7Be line. If the lower ratio measured by the Homestake experiment is real, we expect that the electron-type neutrino flux of the ^7Be line is near a minimum of the oscillation at the average Sun-Earth distance. In other words, there are an odd number of half-wavelengths from Sun to Earth. Then we expect a modulation of the ^7Be neutrino flux with a period of half a year, with maximums occurring at the perihelion and aphelion of the Earth orbit. We see no statistically significant Fourier component of the time dependent Homestake data from 1970.281 to 1994.388.[12] In particular the amplitude relative to the mean of a Fourier component of period 0.5 years is 0.09 ± 0.10. This observation implies that there are ≤ 8.5 periods of oscillation from Sun to Earth at 90% confidence level. With a χ^2

Probability	L/E [km/GeV]
$P(\bar{\nu}_e \to \bar{\nu}_e) > 0.99$	88
$P(\nu_\mu \to \nu_\mu) > 0.99$	0.34
$P(\nu_\mu(\bar{\nu}_\mu) \to \nu_e(\bar{\nu}_e)) < 0.90 \cdot 10^{-3}$	0.31
$P(\nu_\mu \to \nu_\tau) < 0.002$	0.039
$P(\nu_\mu \leftrightarrow \nu_\tau) < 0.35$	31
$P(\nu_\mu \to \nu_\tau) < 0.022$	0.053
$P(\nu_e \to \nu_\tau) < 0.125$	0.031
$P(\nu_e \leftrightarrow \nu_\mu) < 0.25$	40
$P(\bar{\nu}_e \to \bar{\nu}_e) > 0.75[11]$	232

Table 3: Limits on the mixing probabilities from astrophysical, accelerator and reactor experiments.[1]

χ_8^2	$M_2^2 - M_1^2$ [eV^2]	$M_3^2 - M_2^2$ [eV^2]	s_{23}	s_{13}	s_{12}	δ
7.0	$4.9 \cdot 10^{-11}$	$5.0 \cdot 10^{-3}$	0.83	0.08	0.50	-3.14
7.2	$1.6 \cdot 10^{-10}$	$5.0 \cdot 10^{-3}$	0.57	0.00	0.74	-3.14
7.2	$4.3 \cdot 10^{-10}$	$5.0 \cdot 10^{-3}$	0.57	0.00	0.74	3.14

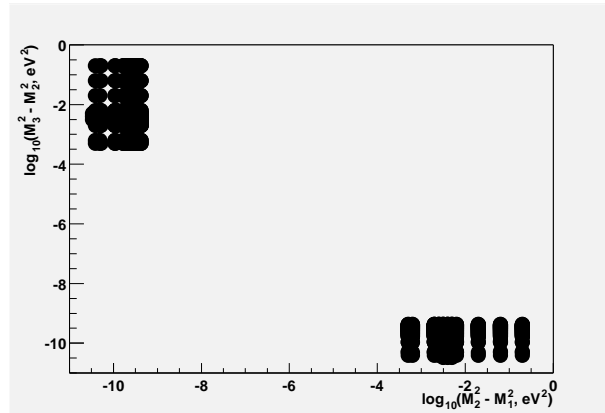Table 4: Parameters at local minima of χ^2 for 8 degrees of freedom.

Figure 1: The mass-squared differences $(M_2^2 - M_1^2, M_3^2 - M_2^2)$ lie within the dots with 90% confidence.
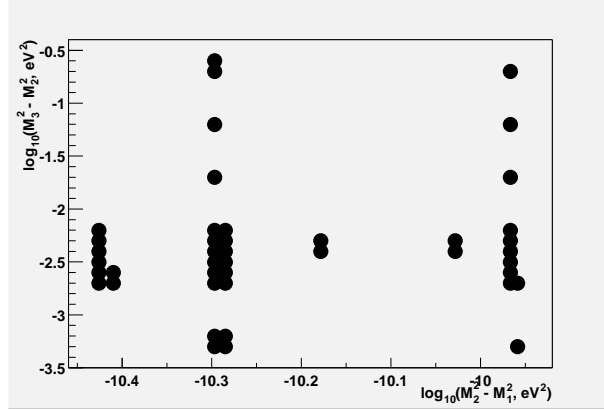
Figure 2: Detail of the "upper island" of Figure 1 for the fit with 116 degrees of freedom (see text) at 90% confidence level. The "lower island" is symmetrical. The vertical bands correspond, from left to right, to 2.5, 3.5, 4.5, 6.5 and 7.5 oscillations from Sun to Earth of the ^7Be line.

with 116 degrees of freedom, including the 8 discussed earlier plus the 108 measurements by the Homestake Collaboration from 1970.281 to 1994.388[12] we obtain the allowed region shown in Figure 2.

The reliability of $M_2^2 - M_1^2$ depends on the correctness of the error assigned to the Homestake observed-to-predicted flux ratio. For example, if the Homestake error listed in Table 1 is doubled we obtain the solutions shown in Figure 3.

In view of the preceeding results let us assume that neutrinos indeed have mass. The question then arizes wether neutrinos are distinct from antineutrinos (Dirac neutrinos) or wether neutrinos are their own antiparticles (Majorana neutrinos). This latter possibility arizes because neutrinos have no electric charge.

Let us consider Big-Bang nucleosynthesis that determines the abundances of the light elements D, ^3He, ^4He and ^7Li. These abundances are determined by the temperatures of freezout $T_f \approx 1 MeV$ when the reaction rates $\propto T_f^5$ become comparable to the expansion rate $\propto T_f^2 \times (5.5 + \frac{7}{4}N_\nu)^{1/2}$. Here N_ν is the equivalent number of massless neutrino flavors that are ultrarelativistic at T_f and are still in thermal equilibrium with photons and electrons at that temperature. The calculated abundances of the light elements are in agreement with observations if $1.6 \leq N_\nu \leq 4.0$ at 95% confidence level.[1] For three generations of Majorana neutrinos, $N_\nu = 3$. For three generations of Dirac neutrinos, $N_\nu = 6$ while in thermal equilibrium. However, in the Standard Model only the left-handed component of neutrinos couple to Z, W^+ and W^-. Right-handed neutrinos are not in thermal equilibrium at
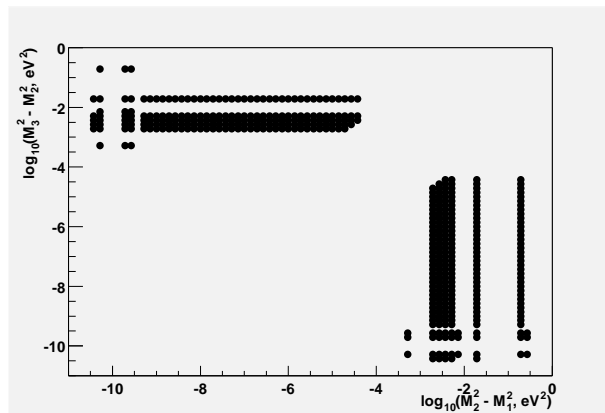
Figure 3: Same as Figure 1 but we have doubled the error of the Homestake experiment, *i.e.* $R = 0.33 \pm 0.10$.

T_f: their temperature has lagged below the temperature of photons due to the anihilation of particle-antiparticle pairs after the decoupling of the right-handed neutrinos. Therefore for Dirac neutrinos at T_f we have $N_\nu \approx 3$. So we can not distinguish Dirac from Majorana neutrinos using available data on nucleosynthesis.

In conclusion, the minimal extension of the Standard Model with three massive Majorana or Dirac neutrinos that mix is in good agreement with all experimental constraints. However, confirmation of the model is needed, *e.g.* by the observation of seasonal variations of the ^7Be spectral lines with a period of 0.5 years, or spectral distortions and seasonal variations of the low energy neutrinos from the solar *pp* reaction.

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